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Absolute Priority Rule Violations and Risk Incentives for Financially Distressed Firms

Allan C. Eberhart and Lemma W. Senbet

Allan C. Eberhart is a Visiting Assistant Professor of Finance at the Stern School of Business, New York University, New York, New York, and an Assistant Professor of Finance at the School of Business, Georgetown University, Washington, D.C. Lemma W. Senbet is the William E. Mayer Professor of Finance at the College of Business and Management, University of Maryland, College Park, Maryland.

■ The role of financial contracting in resolving agency conflicts between creditors and shareholders has been extensively investigated in the finance literature.[1] The purpose of financial contracting is to create a self-enforcing mechanism for the alignment of conflicting interests of securityholders so that shareholders make decisions to maximize firm value.

With rational financial markets, shareholders are forced to absorb any costs that result from unresolved agency conflicts. Hence, it is in their interest to seek contracts — explicit or implicit — that reduce, or even eliminate, agency conflicts. Conversion or call provisions included

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[1]Of course, there can also be agency conflicts between managers and shareholders, but the focus of this paper is on the conflict between creditors and shareholders. Because we assume that management acts in the interests of shareholders, we use the terms "management" and "shareholders" interchangeably. See Barnea, Haugen, and Senbet [6] for a comprehensive analysis of financial agency problems.

in the bond indenture are two well-known examples of the use of explicit financial contracting to mitigate agency conflicts. These provisions are particularly effective in diminishing the incentive for shareholders to shift the firm's assets into high-risk projects. Convertibles mitigate the incentive because equityholders are forced to share the residual payoff with converting bondholders in the event of a large payoff. Call provisions reduce the incentive through a decrease in the underlying bond value (in response to a risk shift) thereby diminishing the value of the call provision held by shareholders. These traditional methods of controlling risk-shifting lose their effectiveness, however, as the firm approaches financial distress, because the probability of a conversion or call approaches zero.

The purpose of this paper is to investigate, in a contingent claims framework, the role of absolute priority rule (APR) violations in reducing agency conflicts between bondholders and shareholders. In particular, we demonstrate that departures from the APR are effective in controlling the risk-shifting incentive of financially distressed firms. The APR states that creditors must be fully compensated before shareholders receive any portion of the bankrupt firm's value.[2] Betker [10], Eberhart. Moore, and Roenfeldt [25], Franks and Torous [28], [29], LoPucki and Whitford [49], and Weiss [60] demonstrate that this rule is enforced in only about 25% of corporate bankruptcy cases. There is also strong evidence that the capital market anticipates departures from the APR (e.g., Betker [11], Eberhart, Moore, and Roenfeldt [25], Eberhart and Sweeney [26], and Warner [59]).

The possibility of a departure from the APR is an implicit feature of the bond contract (implicit because this feature is not explicitly mentioned in the bond covenant at the time of debt issuance) that allows for equity participation on the lower tail of the firm's cash flow distribution. Moreover, because deviations from the APR also occur in informal bankruptcies/workouts (Franks and Torous [29]), these implicit contracts are not dependent on enforcement by the bankruptcy court.

As with explicit financial contracting, APR violations have the attractive feature of self-enforcement. Further, they are most effective when the more traditional methods are least effective; as the probability of a conversion or call approaches zero with a worsening of the firm's financial condition, a deviation from the APR becomes the dominant force in reducing risk-shifting. Thus, an APR violation complements the traditional methods of controlling the risk incentive by serving as an "insurance" policy against their failure when the firm is financially distressed.[3, 4]

The first section discusses the risk-shifting problem and the means of combating the problem. Section II presents the valuation effects of a conversion provision and an APR violation in a contingent claims framework. The impact of convertibles and departures from the APR on the risk incentive are shown in Section III. Section IV concludes with a summary.

I. The Risk-Shifting Problem

The asset substitution problem induced by debt financing has long been recognized in corporate finance. Drawing upon an important insight of Black and Scholes [12] that levered equity can be viewed as a call option on the value of the firm's assets, Galai and Masulis [31] and Jensen and Meckling [44] show that the positive relationship between stock value and underlying asset volatility

[2]The APR also specifies that senior creditors should receive their full contractual payment before junior creditors and there can be these types of APR violations (Betker [10], Eberhart and Sweeney [26], Franks and Torous [28], [29], and Warner [59]). The focus of this study, however, is on APR violations between creditors and shareholders.

[3]Independent work by Berkovitch and Israel [9], Daigle and Maloney [23], Frierman and Viswanath [30] and Gertner and Scharfstein [33] came to our attention after the first draft of this paper was completed in October 1989. All of these papers note that APR violations can reduce the risk-shifting incentive; nevertheless, there are important differences in the analysis. Berkovitch and Israel [9] argue that the automatic stay provision of Chapter 11 creates an incentive for shareholders to reveal information about overinvestment incentives and this leads to renegotiation resulting in deviations from the APR. Daigle and Maloney [23] take a view that APR violations are in the interest of debtholders. Frierman and Viswanath [30] argue that these violations must be enforced by the bankruptcy court. Gertner and Scharfstein [33] mention that departures from the APR can reduce the risk incentive, but do not pursue the issue. Unlike our study, none of these papers demonstrate how APR violations complement the role of the more traditional methods of mitigating the risk incentive.

[4]The intuition behind our argument is straightforward once we recognize that the relative priority rule "contract" has a conversion analog, where conversion can be thought of as exchanging stocks into bonds on the lower tail of the cash flow distributions (i.e., bankruptcy states). The concept of a "convertible stock" seems unorthodox, but it is interesting that a more efficient reduction of the risk-shifting incentive is attained by the use of not only convertible bonds but "convertible stock." Unlike the standard explicit conversion contracts, the "convertibility of stock" here is implicit. Shareholders give up nothing in exchange at the time of conversion, but — as mentioned earlier — the value of the privilege is already accounted for at the time of the bond issuance (i.e., capital structure decision) in the form of a reduction in bond price.

creates an incentive for shareholders to expropriate wealth from bondholders by moving the firm's assets into high-risk projects. Risk-shifting can enhance equity value even when higher risk projects are of lower value, implying that investment decisions can be distorted away from firm value maximization. Rational bondholders recognize this incentive, however, and price the firm's debt with the expectation that shareholders make investment choices towards high-risk (albeit lower value) projects. Hence, the reduction in firm value resulting from the choice of high-risk/low-value projects is an agency cost associated with issuing debt that is absorbed by shareholders.

Because issuing debt creates costs, there is no incentive to issue it unless some benefit is provided. The literature has advanced various countervailing benefits of debt financing. First, the tax code discriminates between debt and equity, whereby debt payments are deductible at a corporate level while equity payments are not. This may give rise to the tax advantage of debt to the extent that the corporate tax deductibility benefit is not outweighed by the disadvantage of taxability of bond income at a personal level (Miller [50]). Second, equity itself has its own costs in the form of informational asymmetries which may be reduced by increased reliance on debt (Jensen and Meckling [44], and Ross [52]). Third, the precommitment nature of the debt contract has been argued to reduce the managerial agency cost associated with the free cash flow problem (Jensen [45]). Finally, debt can arise in optimal security design as a means of minimizing the costs (associated with outside financing) of verifying firm income which could otherwise be appropriated by the inside manager-entrepreneur (Townsend [58]). The intuition is that no verification costs are incurred in the nonbankrupt states when the firm's cash flows are sufficient to meet the fixed debt obligations.

As in the debt agency literature (e.g., Green [39]), we treat the benefits of debt issuance as exogenous to our model and focus on the efficiency gains of financial contracting in reducing debt agency costs.[5] Of course, if there is no additional cost induced by APR violations, our analysis suggests that departures from the APR would lead to greater use of debt.

[5]It is worthwhile to note that we do not see any reason why APR violations would reduce the aforementioned benefits. Even in the case of Townsend [58], departures from the APR do not occur in the nonverification regions, because only the bankruptcy state payoffs are shared with equityholders. Thus, the benefit offered by issuing debt in reducing verification costs should not be affected.

A. Existing Methods of Controlling the Risk Incentive

Jensen and Meckling [44] point out that it can be in the interests of shareholders to ameliorate the risk-shifting concerns of bondholders by restricting managerial actions through bond covenant provisions. These are unlikely to be effective in controlling the risk incentive, though, because they are not self-enforcing.[6] Management has an incentive to cheat to enhance equity value. Moreover, restricting managerial behavior this way can result in foregoing high-risk but highly profitable investment opportunities, leading to non-firm-value-maximizing behavior. In fact, most bond covenants do not contain any restrictions on risk-shifting. Smith and Warner [55] report that only 35.6% of the bond covenants they examine contain restrictions on the disposition of the firm's assets. In a multiperiod framework, Diamond [24] argues that reputation effects reduce management's proclivity to increase risk, but reputation may be less of a concern for managers of financially distressed firms.[7]

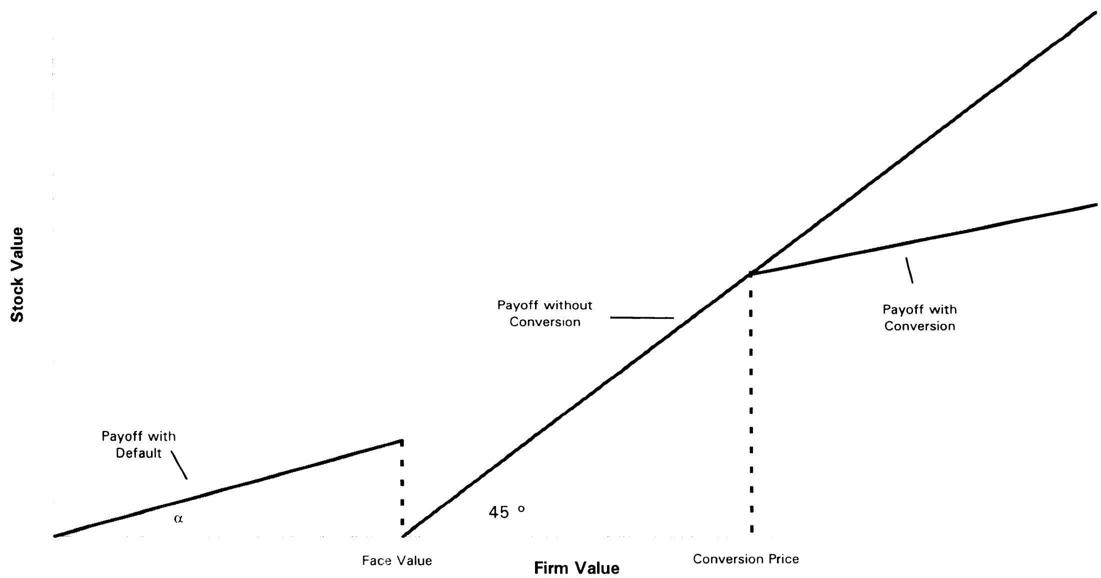
The use of a convertible to mitigate asset substitution is rigorously demonstrated by Green [39]. He shows that by only allowing equityholders to share in the "upper-tail of the upper-tail" of the firm's distributions, a convertible decreases the risk incentive. The key to Green's analysis is that a convertible transforms the shareholder claim from a convex function of firm value to a convex-concave one as shown in Exhibit 1 (the default state payoffs should be temporarily ignored); then the conversion contract can be designed to efficiently trade-off the convex and concave regions to neutralize the risk incentive.

The usefulness of a call provision in dampening the risk incentive is demonstrated by Bodie and Taggart [14] and Barnea, Haugen, and Senbet [5], [6]. The basic argument is that increasing risk reduces the value of the call provision held by shareholders through a reduction in the underlying bond value, thereby offsetting the increase in equity value.

[6]For an analysis of the mitigation of the risk-shifting problem in an asymmetric information context, see John [47]. See Seward [53] for an analysis of the role of financial intermediaries in the mitigation of the risk incentive problem.

[7]A point in favor of the strength of the reputation effect is Gilson's [35] finding that managers of financially distressed firms are more likely to be fired and have a more difficult time getting rehired than managers of nondistressed firms.

Exhibit 1. Payoffs to Shareholders With APR Violation and Use of Convertible Bond



B. APR Violations as Implicit Features of Bond Contracts

Suboptimal risk-shifting incentives provide a strong economic rationale for the existence of explicit financial contracts. By more closely aligning the interests of bondholders and shareholders, they mitigate conflicts of interest between securityholders and hence decrease agency costs. The use of conversion or call provisions, however, is an incomplete way to reduce the shareholder/bondholder agency conflict, because the onset of financial distress makes them virtually worthless. In other words, shareholders will gamble with additional risk because these contracts are no longer effectual in altering the convexity of the equity claim.

It may be possible to renegotiate, say, the conversion price of the convertible, thereby restoring the ability of this mechanism to reduce the risk incentive.[8] Renegotiation of a convertible bond, though, is economically troublesome.[9] It is easy to see in Exhibit 1 that as the conversion price is set closer and closer to the face value of debt (in an attempt to increase the probability of conversion), the equity claim becomes less and less concave. The only way to circumvent this is to lower the face value of the debt, but this is simply a departure from the APR.[10]

This is not to suggest that there are no attempts to renegotiate the implicit contract of an APR violation. Informal and formal bankruptcies inevitably involve each class of securityholders attempting to benefit at the expense of the others. However, as long as the market anticipates the final outcome, then the implicit contract approach is appropriate.

The discussion above raises an interesting question. If APR violations do reduce agency costs, why are they not made explicit? In other words, what is the relative efficiency of having a departure from the APR implicitly featured in the bond contract versus having it explicitly featured? The most plausible answer to this question is that deviation from the APR is situation-specific and the implicit nature of the contract allows for greater flexibility. This view is consistent with Harris and Raviv [41] who

[8] It is important to note that shareholders must have some incentive to renegotiate, such as the need to issue new debt. (*Note:* Without loss of generality, we focus on the failure of conversion provisions to reduce the risk incentive in states of financial distress.)

[9] See Bergman and Callen [7], [8], Brown [16], and Giammarino [34] for game-theoretic models of renegotiation in periods of financial distress.

[10] In the Haugen and Senbet [42, footnote 4] framework, APR violations are a means of internalizing bankruptcy costs in the private system away from the court system.

propose a design of bankruptcy procedures with departures from the APR. They view a contract design as a game to be played later as opposed to contingent allocation schemes. Unlike our paper, though, their notion of APR violations is motivated by the inefficiency of the liquidation decision in states of financial distress.

We cannot rule out the possibility, however, that explicitly featuring an APR violation in the bond contract would be relatively more efficient for some firms. We are merely demonstrating that an APR violation plays a vital role in mitigating, or even eliminating, the problem. Moreover, unlike some unspecified explicit contracts, APR violations are observable phenomena in the institutional mechanisms governing bond contracts.

C. Anticipation of APR Violations: Extant Empirical Evidence

Because APR violations are viewed as implicit features of bond contracts, the effectiveness of these features depends upon a rational anticipation of bankruptcy settlements with relative priority rules. This argument is supported by strong evidence on the ability of the bond and equity markets to forecast the resolution of bankruptcy claims.

Eberhart, Moore, and Roenfeldt [25] analyze 30 bankruptcies that occurred between 1979 and 1986 and find that stock prices, measured around the bankruptcy announcement date, significantly reflect the amount shareholders receive in adherence to the APR and in violation of the rule on the reorganization confirmation — or emergence from bankruptcy — date.

Betker [11] examines 78 firms that filed for Chapter 11 between 1982 and 1990. Though a test of efficiency is not the focus of his study, he reports that the average cumulative abnormal returns (ACAR) for the bankrupt firms' stocks and bonds are generally insignificantly different from zero during the bankruptcy period, consistent with efficiency.

Eberhart and Sweeney [26] test the efficient market hypothesis for a sample of 67 firms that filed for bankruptcy between 1980 and 1990. Their efficiency tests are conducted with the ACAR and price-unbiasedness tests. This latter test asks if the actual return for each bond during the bankruptcy period, cross-sectionally regressed on the bond's expected rate of return, falls along a 45-degree line. They demonstrate that this can be a more powerful test of efficiency than the ACAR test. Overall, their results support efficiency.[11, 12]

II. The Valuation Effect of APR Violations and Convertibles

The effect of a conversion provision and a departure from the APR on stock value can be characterized in the context of the familiar option pricing model derived by Black and Scholes [12]. The model is simple, but yields rich insights. With no conversion provision or APR violation, the stock value is

$$S = VN(d_1) - e^{-rT}BN(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln(V/B) + (r + (\sigma^2/2))T}{\sigma\sqrt{T}},$$
$$d_2 = d_1 - \sigma\sqrt{T},$$

$N\{\cdot\}$ is the cumulative normal density, $\sigma^2 T$ is the variance of the rate of return on firm value (V) over the length of maturity of the debt (T), rT is the known and constant risk-free rate earned over span T, and B is the face value of the firm's zero coupon debt.[13]

With a departure from the APR, shareholders receive a part of the firm value in default states. Assume that $0 \le \alpha < 1$ represents the constant proportion of firm value shareholders receive in violation of the APR. Though it is well known that α exhibits cross-sectional variability (Eberhart, Moore, and Roenfeldt [25]), it is not our purpose here to explain these cross-sectional differences.[14] It is important to note that this is not inconsistent with our earlier argument that APR violations can be situation-specific. The point is that whatever value of $0 < \alpha < 1$ is

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[11] For additional evidence in support of the hypothesis that distressed firms' securities are efficiently priced, see Baldwin and Mason [4], Clark and Weinstein [19], Morse and Shaw [51], and Warner [59].

[12] A recent study by Eberhart and Sweeney [27] notes that although APR violations may not bias distressed security prices, they can introduce additional uncertainty about a security's intrinsic value (i.e., noise). They develop a theoretical model that offers an empirical test of whether APR violations raise the amount of noise in security prices; their empirical results suggest that departures from the APR do increase noise.

[13] We assume a single class of debt. See Stulz and Johnson [57] for an analysis of the role of secured debt in resolving agency conflicts when there is more than one class of debtholders.

[14] Altman, Eberhart, and Zekavat [3] argue that although APR violations can be large in certain cases, on average bonds with high priority provisions receive substantially higher payoffs upon emergence from Chapter 11 than bonds with lower priority provisions (also see Betker [10] and LoPucki and Whitford [49]).

appropriate for the firm, our analysis demonstrates that the risk incentive is reduced.

The present value of the expected magnitude of the APR violation is (see Appendix A for the derivation):

$$Q = \alpha V(1 - N(d_1)). \qquad (2)$$

Under a system of relative priority rules (RPR), and without a conversion provision, the total value of equity (S_R) is the sum of (*i*) the value of equity in nondefault states, and (*ii*) the value in default states (i.e., where $V < B$ at maturity). Therefore, S_R is given by:

$$S_R = S + Q = VN(d_1) - Be^{-rT}N(d_2) + (1 - N(d_1))\alpha V. \qquad (3)$$

Because $N(d_1)$ may be viewed as the probability of avoiding bankruptcy, then $(1 - N(d_1))$ represents the likelihood of the firm going bankrupt. Hence, a departure from the APR increases equity value by the amount given to shareholders in violation of the rule (αV) multiplied by the probability of the firm declaring bankruptcy. The value attributable to the APR violation arises from stockholder participation in the bond payoff and — as suggested earlier — can be viewed as the value of "conversion" into bonds. The first two terms on the right-hand side (RHS) of Equation (3) are the "pure equity" component of stock value, while the last term is the "contingent bond" portion. The addition of a bond component to equity value causes the equity beta to decline (see Appendix B).

If a conversion privilege is attached to the bond, shareholders have sold a call option to bondholders. The resulting bond and equity payoffs are depicted in Exhibit 1. If we assume that this conversion privilege has the same maturity as the debt, then we can simply subtract the value of this option (using the Black-Scholes model) from Equation (3) to obtain the stock value associated with conversion privileges and relative priority rules:

$$S^{**} = VN(d_1) - Be^{-rT}N(d_2) + (1 - N(d_1))\alpha V - \tau(VN(d_1^*) - \frac{B}{\tau}e^{-rT}N(d_2^*)), \qquad (4)$$

where

$d_1^* = \dfrac{\ln(\tau V/B) + (r + (\sigma^2/2))T}{\sigma\sqrt{T}}$,

$d_2^* = d_1^* - \sigma\sqrt{T}$,

τ = conversion ratio,

T = maturity of the conversion,

$\dfrac{B}{\tau}$ = conversion price.

When the firm initially issues debt, the probability of bankruptcy is likely to be small and the valuation impact of a deviation from the APR is accordingly small. Nevertheless, with rational pricing, bondholders pay a fair price for the bonds (i.e., they reduce the bond price by the present value of the expected payoff to shareholders in states of default).

If the APR is followed (i.e., $\alpha = 0$), then S^{**} collapses to

$$S^{*} = VN(d_1) - Be^{-rT}N(d_2) - \tau(VN(d_1^*) - \frac{B}{\tau}e^{-rT}N(d_2^*)). \qquad (5)$$

III. The Risk Incentive[15]

A. The Risk Incentive for a Nondistressed Firm

Assuming the APR holds (and no conversion privilege), a rise in volatility always increases equity value at the expense of bond value. Higher volatility implies a greater chance that shareholders receive a big payoff while limited liability protects them from the concurrent increase in the likelihood of a large loss. An increase in risk reduces bond value, because bondholders do not receive a larger payment than B if a substantial return is generated; conversely, a precipitous loss implies a lower value of their claim in default states.

The ability of a conversion privilege and a departure from the APR to reduce the risk incentive is given by:

$$\frac{\partial S^{**}}{\partial \sigma} = V\sqrt{T}\frac{1}{\sqrt{2\pi}}\exp\frac{-d_1^2}{2} - \alpha V\frac{\partial N(d_1)}{\partial \sigma} - \tau dC, \qquad (6)$$

where

τdC = change in the value of the conversion privilege.

The first term on the right-hand side of Equation (6) is the positive effect of a rise in σ on the pure equity portion of stock value. The second term is the change in the contingent bond portion of stock value. For any empirically reasonable level of σ, $\partial N(d_1)/\partial\sigma$ is positive when the firm

[15]The reaction of the bond price to a change in α is simply the opposite sign of the stock value change. Therefore, to facilitate a parsimonious discussion, only the stock value changes are presented.

is in financial distress (i.e., $V < B$ and $T \leq 1$ year).[16] This causes the beneficial effect of an increase in σ to be abated with an APR violation. In other words, the drop in the contingent bond portion of stock value (in response to an increase in σ) offsets the rise in the pure equity portion. The last term in Equation (6) is the change in the value of the conversion privilege, and because the conversion privilege is a simple call option, τdC is always greater than zero. Therefore, any increase in volatility causes the conversion privilege to offset the increase in the pure equity component of stock value. The dominant role of the conversion privilege when the firm is nondistressed is demonstrated below in a numerical analysis.

The comparative static analysis presented above presumes that V is constant. Of course, as discussed earlier, the reason it is in the interest of shareholders to seek contracts that mitigate the risk incentive is because they are forced to absorb the reduction in firm value from the choice of high-risk/low-value projects.[17] The marginal agency cost of risk-shifting can be thought of as equalling some proportion ($0 \leq X \leq 1$) of the stock value after the rise in volatility; where X depends on the firm's technology (i.e., its investment opportunity set).[18] Let S_1 equal the stock value after the rise in risk, while S_0 represents stock value before the risk shift. Risk-shifting is attractive to shareholders when $S_1 - S_0 > XS_1$. Rearranging terms leads to the following condition for shareholders to increase risk: $S_1 / S_0 > (1 / (1 - X))$. In other words, for a given X, the greater the percentage change in S from an increase in risk, the more attractive the risk shift is for shareholders.

To examine the effect of an APR violation on the risk incentive for a nondistressed firm, a numerical analysis is conducted. In the analysis, $V = \$90$, $B = \$100$, $r = 6\%$, $T = 10$ years, and, for cases where the APR is violated, $\alpha = 8\%$ (Eberhart, Moore, and Roenfeldt [25] report that the average value of α is about 8%). Even though the stock is out-of-the-money, the firm is considered nondistressed because the bond has a zero coupon and there is a long time to maturity. Exhibit 2 shows the stock values for different levels of volatility. Under the APR, and without a conversion privilege (stock value = S), the well-known monotonic relationship between stock value and volatility is illustrated in Exhibit 3. Exhibits 2 and 3 also show that with the inclusion of a conversion privilege in the bond covenant, but no APR violation (stock value = S^*), there can actually be a negative relationship between stock value and volatility. This only holds for low values of σ, though. The more important point is that the change in stock value (as well as the percentage change) in response to a change in volatility is always less when a convertible bond is used; this illustrates the ameliorating influence of convertibles on the risk incentive.

As expected, a departure from the APR has a small valuation impact (stock value = S^{**}); with $\sigma = 0.1$, the value of this payoff involves only about one percent of the total stock value ((\$33.47 − \$33.16)/\$33.47). Nevertheless, the value is accurately reflected in security prices. Exhibit 3 illustrates the relationship between S^{**} and σ; it is qualitatively the same as the relationship between S^* and σ. Because the APR violation has an economically insignificant effect on the risk incentive in this nondistressed state, the effect of a change in σ on stock value is only shown for S and S^*.

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[16]It is possible for $\partial N(d_1) / \partial\sigma$ to be negative when the firm is nondistressed, leading to the technical possibility that APR violations exacerbate the risk incentive. However, the valuation effect of APR violations is so small in this state that it can be viewed as economically insignificant. When the firm is financially distressed and the valuation impact of APR violations is economically significant, $\partial N(d_1) / \partial\sigma$ is greater than zero when $\sigma^2 > 2r + (\ln(V/B)/T)$ (condition (*)). With $V < B$, this implies that $\ln(V/B)/T < 0$. From an empirical point of view, σ^2 is likely to be greater than $2r$.

Aharony, Jones, and Swary [2] report an average standard deviation of around 0.7 for the equity of 45 firms in the year preceding their bankruptcy. The standard deviation of the firm value (σ) equals the standard deviation of stock value (σ_e) divided by the elasticity of stock value with respect to firm value (ε). In the simulation analyses, ε hovers around the value of 1 (when $V < B$). If $\varepsilon = 1.5$, this implies $\sigma = 0.46$ (when $\sigma_e = 0.7$).

The average rate for three-month Treasury bills was about six percent during the period covered by Aharony, Jones, and Swary's sample (1970-1978). These values imply that σ^2 is greater than $2r$ (i.e., $\sigma^2 = 0.21$ and $2r = 0.12$). The addition of $(\ln(V/B)/T)$ to the RHS of condition (*) serves to reinforce the plausibility of the condition holding.

[17]The implied relationship between V and the capital structure is technically a violation of the model's assumptions; thus, the insights to be gained in this option pricing framework are inherently qualitative (see Jensen and Meckling [44] and Long [48]). Despite this limitation, our model provides useful insights because it accurately captures the benefit of the risk incentive. The benefit to shareholders from choosing a high-risk/low-value project is the rise in equity value; the cost is the lower firm value. By demonstrating a reduction in the marginal benefit (and with no reason to expect a change in the marginal cost), we illustrate the value of convertibles and APR violations in reducing the incentive to raise risk.

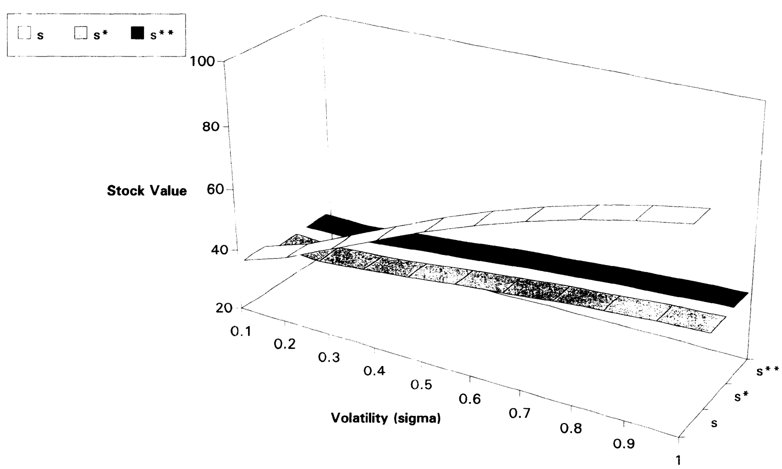
[18]Gavish and Kalay [32] argue that the marginal agency costs associated with risk-shifting do not increase monotonically with higher debt levels. However, Green and Talmor [40] present a generalization of the Gavish and Kalay model and argue the asset substitution problem is exacerbated with greater levels of debt. See Barnea, Haugen, and Senbet [6] for some detailed discussions of agency cost functions.

Exhibit 2. Numerical Analysis for a Nondistressed Firm

T	σ	S	S^*	S^{**}
10	0.1	$35.67550	$33.15951	$33.46557
	0.2	40.47397	32.43802	33.41747
	0.3	47.15772	33.51627	34.66605
	0.4	54.02042	35.09135	36.19322
	0.5	60.49085	36.74471	37.71618
	0.6	66.32046	38.30813	39.12360
	0.7	71.39868	39.70632	40.36670
	0.8	75.69446	40.90812	41.42714
	0.9	79.23020	41.90766	42.30477
	1.0	82.06453	42.71475	43.01099

Note: This exhibit reports the numerical analysis results when the firm is nondistressed (i.e., $V < B$ and $T > 1$). The values for the independent variables are: $\tau = 0.5$, $\alpha = 8\%$, $r = 6\%$, $V = \$90$, $B = \$100$. S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR.

Exhibit 3. The Relationship Between Stock Value and Volatility With Ten Years to Maturity



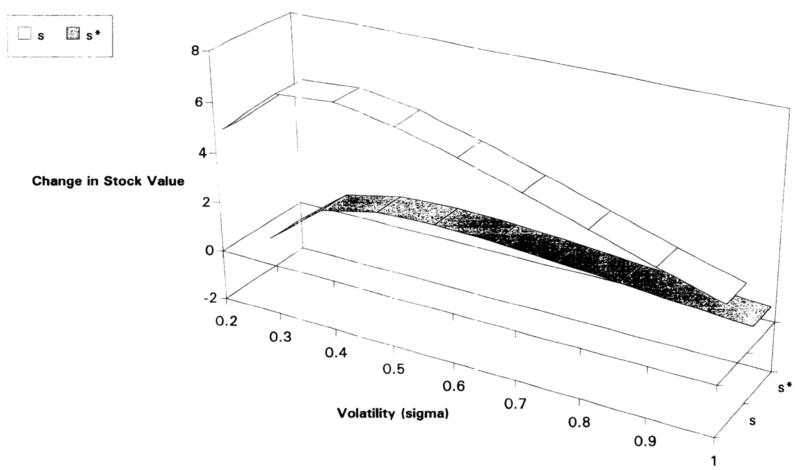
Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR ($r = 6\%$, $B = \$100$, $V = \$90$, $\alpha = 8\%$, and $\tau = 0.5$).

Exhibit 4 shows the relationship between changes in stock value and volatility (Exhibit 5 shows the percentage changes in stock value and volatility). Both exhibits demonstrate the ameliorating influence of a convertible on the risk incentive; that is, the change (and percentage change) in S^* is lower than the change (and percentage change) in S throughout the entire range of σ.

B. The Risk Incentive With Financial Distress

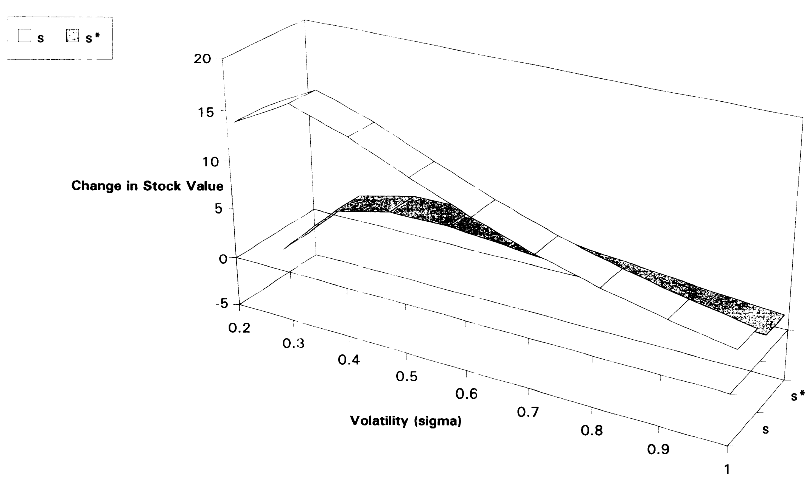
If the firm becomes financially distressed, the effectiveness of the convertible declines dramatically. Moreover, the temptation for shareholders to invest in risky projects intensifies as the firm's financial distress worsens. With the assumption of adherence to the APR, Golbe [37], [38] draws upon the work of Bulow and Shoven [18] to show that the firm facing financial distress may have a greater incentive to invest in riskier projects than firms confident of continuation. Although Golbe does not consider the use of convertible bonds, we demonstrate in the numerical analysis below that a convertible instrument is ineffective when the firm is financially distressed. In this situation, it is critical that the APR be violated for the risk incentive to be significantly reduced.

Exhibit 4. The Change in Stock Value (in Response to a Change in σ) for Each Level of Volatility With Ten Years to Maturity



Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR (r = 6%, B = \$100, V = \$90, and τ = 0.5).

Exhibit 5. The Percentage Change in Stock Value (in Response to a Change in σ) for Each Level of Volatility With Ten Years to Maturity



Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR (r = 6%, B = \$100, V = \$90, and τ = 0.5).

As mentioned earlier, financial distress occurs when $V < B$ and $T \leq 1$ year. We arbitrarily require $T \leq 1$, because when the pure discount bond has a long time to maturity, it is not plausible to assert that the firm is financially distressed simply because the stock is out-of-the-money.

For the financial distress numerical analysis, V = \$90, B = \$100, r = 6% and α = 8% (in those cases where the APR is violated). These are the same values used for the nondistressed scenario; the only difference is that $T \leq 1$ year. The stock is now out-of-the-money with a short time until maturity. The effect of a change in σ is examined for T = 1, 0.5 and 0.1 year — or just before the firm would declare bankruptcy. Because $V < B$, the effect of worsening financial distress on the risk incentive is simulated by examining ∂(stock value) / $\partial\sigma$ with lower and lower values of T. In other words, as $T \rightarrow 0$, the firm can be loosely (recall that this is a one-period model) viewed as approaching bankruptcy.

It should be emphasized that in our model, default occurs when firm value is less than the amount owed to pure discount bondholders at the maturity date of the debt. Of course, many firms in default do not enter formal bankruptcy proceedings (see Berkovitch and Israel [9], Brown, James, and Mooradian [17], Franks and Torous [29], Gilson, John, and Lang [36], Haugen and Senbet [42], [43], and Jensen [46] for a discussion of informal bankruptcies/workouts). In the context of our simple one-period model, though, it is not important whether the firm reorganizes formally or informally. The two critical assumptions are: (*i*) in default states, shareholders receive some compensation even though creditors are not fully compensated; and (*ii*) the capital market rationally anticipates that shareholders will be given a payment in violation of the APR. There is empirical support for both of these assumptions.

First, there is documentation of departures from the APR in workouts and formal bankruptcies (see work cited earlier). Second, there is strong evidence on the ability of the capital market to rationally anticipate the resolution of workouts (Gilson, John, and Lang [36]) and formal bankruptcies (see work cited earlier).[19] Thus, when we show the effect of an APR violation on risk-shifting as the firm nears bankruptcy, the firm can be viewed as approaching a formal or informal bankruptcy.[20]

Exhibit 6 gives the numerical analysis results of the effect of volatility changes on the stock value when the firm is financially distressed. With T = 1 and σ = 0.1, there is essentially no difference — when the APR is followed — in the stock value *with* versus *without* the conversion privilege; this underscores the point that the onset of financial distress attenuates the effectiveness of convertibles in reducing the risk incentive. As σ rises, the conversion provision becomes more important, but it is always less effective than the implicit feature of a departure from the APR in the bond contract (i.e., an examination of Exhibit 6 reveals that the change in stock value is always lower when the APR is violated). With a decline in T (see Exhibits 6 through 8), the difference between S and S^* becomes even smaller; in fact, with T = 0.1, S and S^* are virtually identical throughout the entire range of σ.

The percentage changes in stock values are shown in Exhibits 9 through 11. With T = 1, an increase in σ from 0.4 to 0.5 causes S to increase 28.27%, S^* to rise 23.67% and S^{**} to increase 17.26%. When T drops to 0.5 (Exhibit 9), the same increase in σ causes S to rise 34.36%, S^* increases 33.04%, and S^{**} rises 19.23%. Finally, when the firm is on the verge of default (T = 0.1 — see Exhibit 9) the same change in σ results in a 63.14% jump in S and S^* whereas S^{**} only increases 7.52%.[21]

These calculations underscore two points: First, as the bankruptcy date approaches, the conversion provision loses virtually all of its effectiveness in reducing the risk incentive. At T = 1, the difference in the percentage change in S versus S^* is 4.53 percentage points (28.27% − 23.67%). When T = 0.5, this difference drops to 1.54 percentage points, and at T = 0.1, the difference is essentially zero. Second, the effectiveness of a deviation from the APR in mitigating risk-shifting increases as T declines, or as the firm approaches bankruptcy. At T = 1, the difference in the percentage change in S^* versus S^{**} is 6.41 percentage points (23.67% − 17.26%). When T = 0.5, this difference increases to 13.81 percentage points, and at T = 0.1, the difference balloons to 55.62 percentage points. Thus, as the more traditional mechanisms lose effective-

[19]Franks and Torous [29] find about nine percent of the firm value (in workouts) is given to shareholders in violation of the APR. Gilson, John, and Lang [36] report that negative stock price reactions to debt restructuring announcements are more precipitous for firms whose restructuring attempts ultimately fail.

[20]This raises a policy issue on the efficiency of Chapter 11 of the bankruptcy code. Because the beneficial agency effects of APR violations occur in our model whether the firm enters a formal or informal bankruptcy, the results from this paper should not be viewed as an argument for the efficiency of Chapter 11, which is a separate issue that we do not address in this study. There are, however, a number of studies that discuss the role of priority rules and the efficiency of the bankruptcy code (e.g., Adler [1], Bradley and Rosenzweig [15], Harris and Raviv [41], and White [61]).

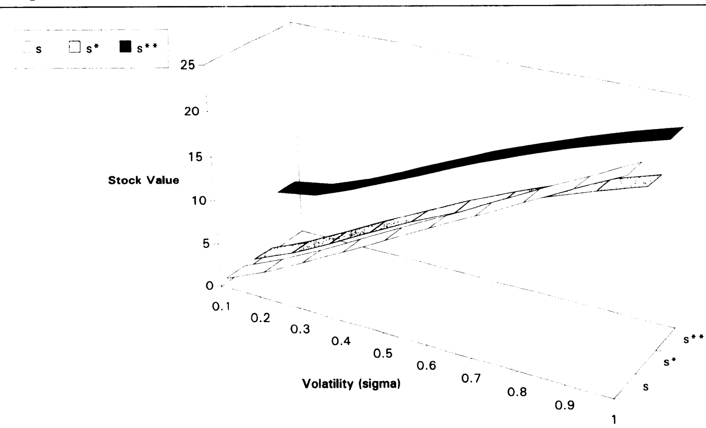
[21]The percentage changes in stock value in Exhibit 9 are lower with higher levels of σ because the stock value base increases as σ rises. For example, with S (at T = 1) an increase in σ from 0.1 to 0.2 causes a 177.96% increase in the stock value from \$1.955 to \$5.434; an increase in σ from 0.2 to 0.3 leads to a 65.91% increase in the stock value from \$5.434 to \$9.016.

Exhibit 6. Numerical Analysis for a Distressed Firm

T	σ	S	S^*	S^{**}
0.1	0.1	\$0.000681	\$0.000681	\$7.194307
	0.2	0.148764	0.148764	6.903420
	0.3	0.682088	0.682088	6.739615
	0.4	1.473536	1.473536	6.981050
	0.5	2.403900	2.403899	7.505930
	0.6	3.412453	3.412413	8.207356
	0.7	4.467848	4.467229	9.020391
	0.8	5.552758	5.548780	9.904622
	0.9	6.656939	6.641824	10.831630
	1.0	7.773821	7.732671	11.779170
0.5	0.1	0.484819	0.484819	6.594652
	0.2	2.479456	2.479456	7.361171
	0.3	4.868794	4.868297	9.176681
	0.4	7.364044	7.345472	11.303600
	0.5	9.894249	9.772756	13.477180
	0.6	12.431520	12.049230	15.550180
	0.7	14.964570	14.134000	17.460900
	0.8	17.484060	16.024700	19.196560
	0.9	19.984120	17.739130	20.769120
	1.0	22.460100	19.300440	22.198210
1.0	0.1	1.955113	1.955113	6.683673
	0.2	5.434372	5.434029	9.397242
	0.3	9.016361	8.972729	12.576140
	0.4	12.601090	12.275540	15.627150
	0.5	16.163430	15.180710	18.325160
	0.6	19.687590	17.693120	20.653960
	0.7	23.162700	19.879560	22.671360
	0.8	26.579610	21.808300	24.441270
	0.9	29.930240	23.534040	26.016090
	1.0	33.207300	25.097570	27.435320

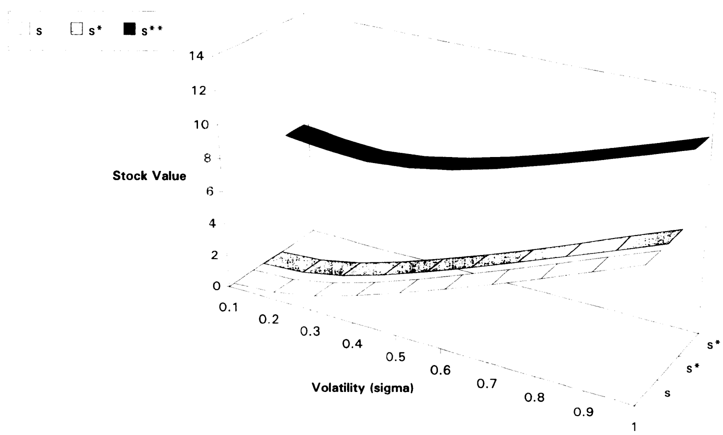
Note: This exhibit reports the numerical analysis results when the firm is financially distressed (i.e., $V < B$ and $T \leq 1$). The values for the independent variables are: $\tau = 0.5$, $\alpha = 8\%$, $r = 6\%$, $V = \$90$, and $B = \$100$. S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR.

Exhibit 7. The Relationship Between Stock Value and Volatility With 0.5 Years to Maturity



Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR ($r = 6\%$, $B = \$100$, $V = \$90$, $\alpha = 8\%$, and $\tau = 0.5$).

Exhibit 8. The Relationship Between Stock Value and Volatility With 0.1 Years to Maturity



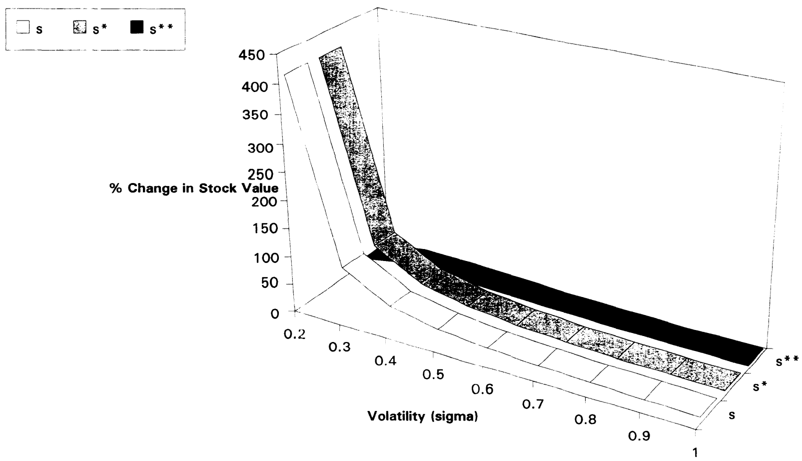
Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR (r = 6%, B = \$100, V = \$90, α = 8%, and τ = 0.5).

Exhibit 9. Numerical Analysis for a Financially Distressed Firm

T	σ	dS/S	dS^*/S^*	dS^{**}/S^{**}
0.1	0.2	217.23599000	217.23599000	-0.04043290
	0.3	3.58502157	3.58502157	-0.02372800
	0.4	1.16032836	1.16032836	0.03582326
	0.5	0.63138192	0.63138124	0.07518639
	0.6	0.41954865	0.41953260	0.09344957
	0.7	0.30927751	0.30911147	0.09906174
	0.8	0.24282607	0.24210780	0.09802579
	0.9	0.19885271	0.19698816	0.09359347
	1.0	0.16777711	0.16423907	0.08747898
0.5	0.2	4.11418478	4.11418478	0.11623342
	0.3	0.96365412	0.96345367	0.24663331
	0.4	0.51249857	0.50883810	0.23177432
	0.5	0.34358906	0.33044629	0.19229095
	0.6	0.25643896	0.23294084	0.15381556
	0.7	0.20376028	0.17302101	0.12287446
	0.8	0.16836367	0.13376963	0.09940266
	0.9	0.14299081	0.10698671	0.08191884
	1.0	0.12389737	0.08801502	0.06880840
1.0	0.2	1.77956926	1.77939382	0.40599966
	0.3	0.65913577	0.65121109	0.33827989
	0.4	0.39758046	0.36809436	0.24260305
	0.5	0.28270094	0.23666331	0.17264888
	0.6	0.21803292	0.16550016	0.12708211
	0.7	0.17651271	0.12357571	0.09767618
	0.8	0.14751777	0.09702126	0.07806810
	0.9	0.12606016	0.07913225	0.06443282
	1.0	0.10948993	0.06643695	0.05455200

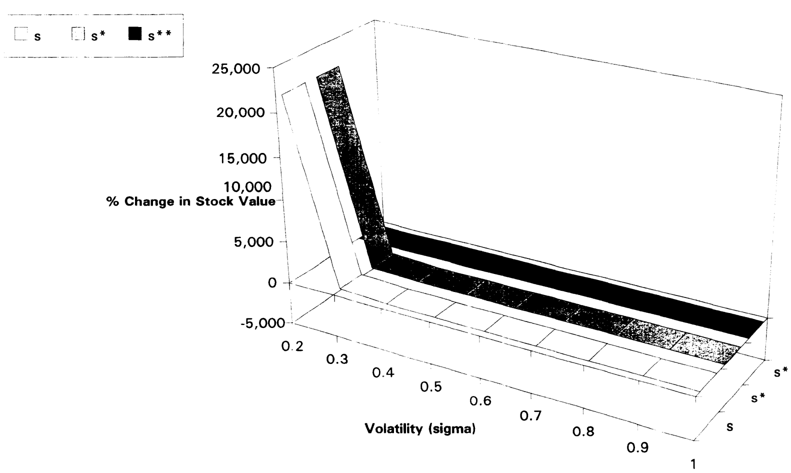
Note: This exhibit reports the numerical analysis results when the firm is financially distressed (i.e., $V < B$ and $T \leq 1$). The values for the independent variables are: τ = 0.5, α = 8%, r = 6%, V = \$90, B = \$100. dS/S = ratio of the change in stock value to stock value with APR and no convertible (in response to an increase in volatility); dS^*/S^* = ratio of the change in stock value to stock value with convertible and APR (in response to an increase in volatility); dS^{**}/S^{**} = ratio of the change in stock value to stock value with convertible and departure from the APR (in response to an increase in volatility).

Exhibit 10. The Percentage Change in Stock Value (in Response to a Change in σ) for Each Level of Volatility With 0.5 Years to Maturity



Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR (r = 6%, B = \$100, V = \$90, α = 8%, and τ = 0.5).

Exhibit 11. The Percentage Change in Stock Value (in Response to a Change in σ) for Each Level of Volatility With 0.1 Years to Maturity



Note: S = stock value with APR and no convertible; S^* = stock value with convertible and APR; S^{**} = stock value with convertible and departure from the APR (r = 6%, B = \$100, V = \$90, α = 8%, and τ = 0.5).

ness in the vicinity of financial distress, the impact of a departure from the APR becomes more pronounced in combating the risk incentive problem.

IV. Conclusions

Recent studies have shown that adherence to the absolute priority rule (APR) seldom occurs in corporate bankruptcies. With a departure from the APR, the levered stock can no longer be viewed as a call option with a simple convex payoff where shareholders walk away from the firm in states of default. This has interesting implications for the investment incentives of managers working on behalf of shareholders.

This paper demonstrates, in a contingent claims framework, that APR violations play an important role in ameliorating the shareholder/bondholder agency conflict. Specifically, a departure from the APR mitigates the incentive for shareholders to increase the risk of the financially distressed firm. The condition of financial distress is of special interest, because this is when the more traditional methods of controlling the risk-shifting incentive (e.g., use of convertible bonds) are ineffective. Hence, an APR violation complements these traditional methods by serving as an "insurance" policy against their failure when the firm is in financial distress. Moreover, the effectiveness of this policy increases as the firm approaches bankruptcy. These results are reinforced and corroborated by our numerical analysis.

It is important to recognize that deviations from the APR also occur in informal bankruptcies, and hence the insights derived from this study apply equally well to firms that do not end up in bankruptcy court. Our analysis does not have direct policy implications on the efficiency of the bankruptcy code, because APR violations do not appear to be brought about solely by outside forces such as the court system. Indeed, departures from the APR may occur as a consequence of bondholders and shareholders being able to internalize bankruptcy costs away from the court system. The policy issues regarding the bankruptcy code and its bearing on APR violations warrant further research.

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Appendix A. Stock Value Model Derivation

In perfect capital markets, the current value of the firm (V) is expressed as the sum of the current market value of its debt (D) and equity (S). The promised payment to debtholders (B) is due at the end of period T. The value of the firm at the end of period T is denoted V_T and is assumed to be a log-normal random variable. Investors are assumed to be risk-neutral and have homogeneous beliefs about the relevant valuation parameters.[22] The debt and equity functions are derived using the following theorem presented in Smith [54, p. 292]:

Theorem:[23] Let the payoffs to shareholders across all states of nature be represented by P (where Φ, φ, Γ and Ω are arbitrary parameters):

[22] The assumption of risk-neutrality is not necessary. Cox and Ross [20] and Cox [21] point out that because the Black-Scholes option pricing model can be derived without any preference assumption, then it must hold for any particular preference assumption. As Smith [54] notes, assuming risk-neutrality merely serves to simplify the derivation.

[23] As Smith notes, the proof of this theorem follows the proof of a less general result in Sprenkle [56].

$$P = \begin{cases} 0, & \text{if } V_T > \varphi B, \\ \Gamma V - \Omega B, & \text{if } \varphi B \geq V_T \geq \Phi B, \\ 0, & \text{if } V_T < \Phi B. \end{cases}$$

Then, the expected value of the payoffs to shareholders will be:

$$E(P) = \int_{\Phi B}^{\varphi B} (\Gamma V_T - \Omega B) L(V_T) dV, \qquad (A1)$$

where $L(V_T)$ is the log-normal density function. Equation (A1) may be rewritten as:

$$E(P) = e^{rT}\Gamma V \left\{ N\left[\frac{\ln(V/\Phi B) + (r + (\sigma^2/2))T}{\sigma\sqrt{T}}\right] - N\left[\frac{\ln(V/\Phi B) + (r + (\sigma^2/2))T}{\sigma\sqrt{T}}\right]\right\} - \Omega B\left\{ N\left[\frac{\ln(V/\Phi B) + (r - (\sigma^2/2))T}{\sigma\sqrt{T}}\right] - N\left[\frac{\ln(V/\Phi B) + (r - (\sigma^2/2))T}{\sigma\sqrt{T}}\right]\right\}. \qquad (A2)$$

The theorem describes how to evaluate the integral in Equation (A1), employing tabulated values of the cumulative normal density, $N\{\cdot\}$.

If the APR holds, the value of the stock is determined from Equation (A2) by setting $\varphi = \infty$, $\Phi = 1$, $\Gamma = 1$, and $\Omega = 1$. After making these substitutions in Equation (A2) and simplifying, the expected payoff to equityholders at the end of period T is:

$$E(P) = e^{rT} VN(d_1) - BN(d_2), \qquad (A3)$$

The present value of $E(P)$, hence the value of equity, is given by:

$$S = VN(d_1) - e^{-rT} BN(d_2). \qquad (A4)$$

This, of course, is the familiar call option formula derived by Black and Scholes [12] as applied to stock valuation.[24]

If the APR is violated, equityholders will also receive a portion of firm value in default states, i.e., where $V_T < B$. Thus, the present value in Equation (A4) will be increased by the present value of the equity claims in default. The expected payoff in default is:

$$E(P) = \int_0^B \alpha V_T L(V_T) dV. \qquad (A5)$$

The theorem above may now be employed to solve the integral in Equation (A5). Letting $\Phi = 0$, $\varphi = 1$, $\Gamma = \alpha$, and $\Omega = 0$, Equation (A2) becomes:

$$E(P) = e^{rT}\alpha V(1 - N(d_1)). \qquad (A6)$$

The present value of $E(P)$, continuing the assumption of risk neutrality, is given by:

$$Q = \alpha V(1 - N(d_1)). \qquad (A7)$$

Therefore, S_R is given by:

$$S_R = VN(d_1) - Be^{-rT}N(d_2) + (1 - N(d_1))\alpha V. \qquad (A8)$$

Appendix B. APR Violations and Beta

The stock valuation model presented in Equation (3) contains a measure of total risk (σ) but is silent with respect to systematic risk. Black and Scholes [12] show that combining the intertemporal capital asset pricing model with their option pricing model leads to the following relationship:

$$\beta_s = \varepsilon\beta_V, \qquad (B1)$$

where β_s is the equity beta, ε is the elasticity of stock value with respect to firm value, and β_V is the firm value, or asset, beta. A comparison of the elasticity of stock value under adherence to the APR (ε_{apr}) with the elasticity under relative priority rules (ε_R) reveals that $\varepsilon_{apr} > \varepsilon_R$ over all values of V and S. Hence, for any given β_V, a deviation from the APR causes the systematic risk of stock to decline.[25]

[24] In addition to the log-normal diffusion process assumed here, Equation (A4) can be derived assuming V follows other stochastic processes (see Cox and Ross [20] and Cox [21]).

[25] The proof of this property of the model is available from the authors.